Modulus of Subgrade Reaction - Which One Should be Used?

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The modulus of subgrade reaction is an often misunderstood and misused concept for the thickness design of slabs-on-ground. Terzaghi in 1955 (Ref. 1, P. 300) attributed this confusion to the initial work by Hayashi in 1921 (Ref. 2), who stated that the subgrade reaction should be determined by a load test but failed to mention that the tested results depend on the size of the loaded area. Additionally, Terzaghi noted that the well-known book by Hetenyi *Beams on Elastic Foundation* (Ref. 3), published in 1946, did not contain any statement regarding the adjustment factors necessary to the subgrade reaction value for the different loading conditions. As stated by Terzaghi, "This condition led to the erroneous conception, widespread among engineers, that the numerical value of the coefficient of subgrade reaction depends exclusively on the nature of the subgrade. In other words, it became customary to assume that this coefficient has a definite value for any given subgrade." Terzaghi had hoped his paper in 1955 would finally clear up this confusion and provide the necessary factors for making the appropriate adjustments to the subgrade reaction load test value. It has been over 60 years since his paper has been published, and we still often see engineers using inappropriate values for the subgrade reaction. Even with the many well-known text books that discuss the issue of subgrade reaction adjustment factors (Ref. 4, P. 286 & 489; Ref. 5, P. 516-517; Ref. 6, P. 501-503; and Ref. 7, P. 416-417), this issue is still frequently not being properly addressed. We hope, by providing detailed examples and specific subgrade reaction values for different soil types for industrial slab loadings, that this approach will provide clarity to this issue.

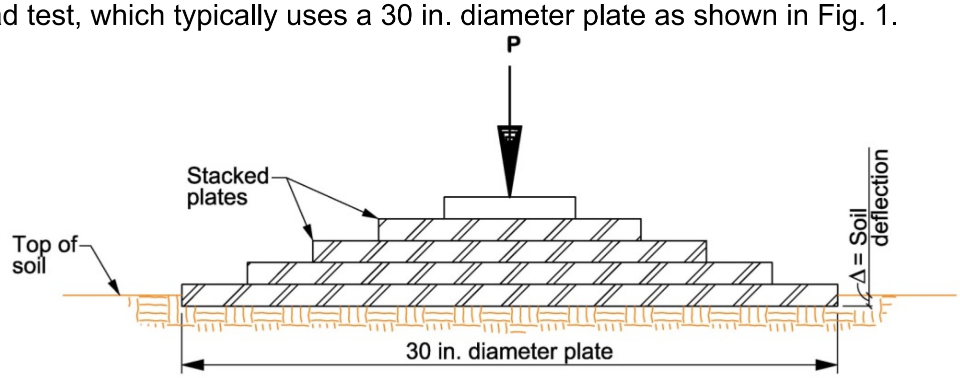
What is the modulus of subgrade reaction?

To add to the confusion, especially for the novice slab designer, this value has been called by many different names in various publications such as *modulus of subgrade reaction*, *subgrade reaction*, *subgrade modulus, coefficient of subgrade reaction*, *Winkler foundation* (E. Winkler first proposed the subgrade reaction in 1867 [Ref. 16]) , *Winkler subgrade*, "*K*" value, etc.. In our paper, we will use either *subgrade reaction* or the shortened "*K*" for our discussions.

The basic equation is:

$$K = \frac{pressure}{deflection} = \frac{\sigma}{\Delta}$$

which has units of pounds/in^2/in or commonly used units of pounds/in^3 or pci. There is an ASTM D1196 (Ref. 8) for the plate load test, which typically uses a 30 in. diameter plate as shown in Fig. 1.



$$\sigma = \frac{P}{\pi R^2}$$

Fig. 1 ASTM D1196 Plate Load Test

In our paper, we will refer to K_p as the subgrade reaction based on the 30 in. diameter plate load test.

The subgrade reaction is not a fundamental soil property. It is a lump constant of which the subgrade reaction from the plate load test should be adjusted because the subgrade reaction is a function of:

1. Soil elastic properties, both the initial response and the long-term response due to soil consolidation from the sustained loading.
2. Loading intensity that will influence the long-term consolidation settlement.
3. Amount of surface area loaded and load shape over which the load is applied. Wider and larger area loadings will involve consolidation of the deeper soil layers.
4. Stiffness of the slab, which will influence the distribution of the soil bearing pressure.

Additionally, using the subgrade reaction (Winkler foundation) has other analytical limitations. The displacement at one location does not influence the settlements at other locations, which is not correct as shown in Fig. 2.

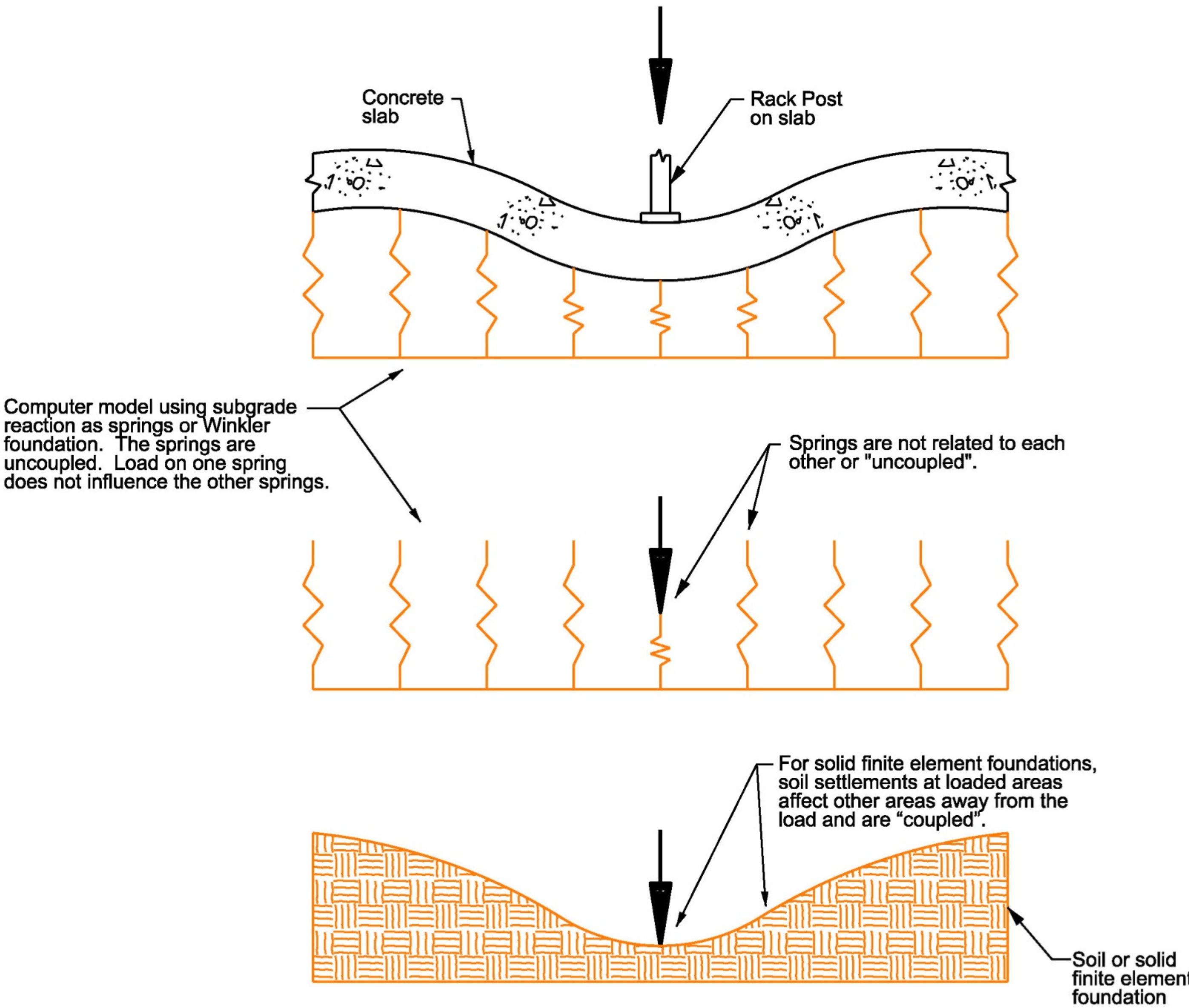


Fig. 2 Uncoupled and coupled analytical foundation systems

Using the subgrade reaction soil springs (Winkler foundation) has been referred to as an “uncoupled” foundation because the springs do not interact with each other. Whereas using the solid finite element foundation, the soil settlements are “coupled”. In other words, the soil settlements at the loaded areas also cause other unloaded areas to settle as well. To improve the accuracy of the subgrade reaction adjustment factors, we have equated the slab stresses for a slab on a solid finite element foundation (coupled model) with a slab using an equivalent subgrade reaction (uncoupled model), as shown in Fig. 5 and values in Table 1. This equalization will adjust the subgrade reaction value for wide area loading effects, consolidation of the deeper layers of the soil, and help minimize the uncoupled analytical limitation of the subgrade reaction foundation system.

Who makes the adjustments to the subgrade reaction values?

Along with the widespread misconception that the subgrade reaction is a single value, this issue is further complicated because it is not clear in the industry who makes the adjustments to the subgrade reaction for the different load cases. The structural engineer often sees a single value in the geotechnical report that typically does not have any qualifiers regarding how the stated subgrade reactions is to be used and assumes (or more likely is uninformed on the limitations on using the subgrade reaction foundation) the geotechnical engineer has made the adjustments to the provided subgrade reaction. The geotechnical engineer typically provides in the geotechnical report the value to use for wheel loadings but often does not provide any qualifiers or limits on how the subgrade reaction is to be used. The geotechnical engineer assumes (or more likely did not consider the limits of the provided subgrade reaction) the structural engineer will make the adjustments based on the different loading cases. As you can see from the scenario above, there is much confusion regarding who is responsible for making the adjustments to the subgrade reaction for the different load cases.

In our opinion, the responsibility is shared jointly between the structural engineer and the geotechnical engineer for determining the proper subgrade reaction to be used for the different load cases. The structural engineer has the responsibility to:

1. Provide the load intensity on the slab and the type of load (such as lift-truck or other vehicle loadings, long-term uniform storage or rack loads, line or wall loads, etc.)
2. Provide the amount and size of slab surface areas that will be loaded and the length of time the load will be applied. It is important that the geotechnical engineer know if the slab will have a wide area loading. Wide area loadings will cause deeper soil consolidation settlements (assuming long-term settlements are significant for the project soil types) if the loads are sustained for a long period of time.
3. Provide the owner’s and/or equipment’s sensitivity to slab cracking, soil settlements (initial and long-term consolidation), slab sloping due to soil settlements, etc.

The geotechnical engineer has the responsibility to:

1. Use the information provided by the structural engineer and the site soil properties to provide recommendations for subgrade reactions to be used for the different load cases.
2. Recommend any testing that needs to be performed to support the provided subgrade reactions values.

Typically for industrial slabs, the geotechnical engineer should provide two subgrade reactions. The first subgrade reaction is for short-term loadings to be used for loads such as lift-truck wheel loads. These loads only affect the first few feet of the soil and have little, if any, long-term consolidation settlements. The second subgrade reaction is for long-term loadings to be used for loads such as wide area rack or uniform storage loads. These loads affect many feet of soil below the slab and have both initial and possible long-term soil consolidation settlements (depending on the soil type). These subgrade reaction values should be stated on the structural project drawing general notes design criteria. ACI 360R (Ref. 9, P. 67) has provided an example design criteria showing how these subgrade reaction values should be shown on the drawings.

Subgrade reaction values for short-term loads over small areas, such as wheel loads

For short duration loadings over small areas, such as wheel loads from lift trucks, where only a few feet of soil is affected below the slab and where long-term soil consolidation is not a concern, the subgrade reaction value from the plate load test should be used. Because these types of loads only affect the first few feet of soil, the subgrade reaction can be improved or increased with a granular aggregate or cement-treated (or other stabilizing materials) base material (Ref's. 10 and 11). Tables and charts for these increased subgrade reaction values using base materials can be found in a number of publications (Ref. 12, P. 6; Ref. 13, P. 6; Ref. 14, P. 3).

Subgrade reaction values for wide area long-term sustained loads, such as uniform loading or rack loads

Determining the appropriate subgrade reaction values for wide area sustained loadings is more complicated than for short-term loadings. These loads involve deeper layers of soil that may or may not have significant long-term consolidation settlements, depending on the soil type. Additionally, as discussed by Terzaghi (Ref. 1, P. 306), the soil settlements used to adjust the plate load subgrade reaction are the settlements above the nearly uniform pressure elevation, as shown in Fig. 3.

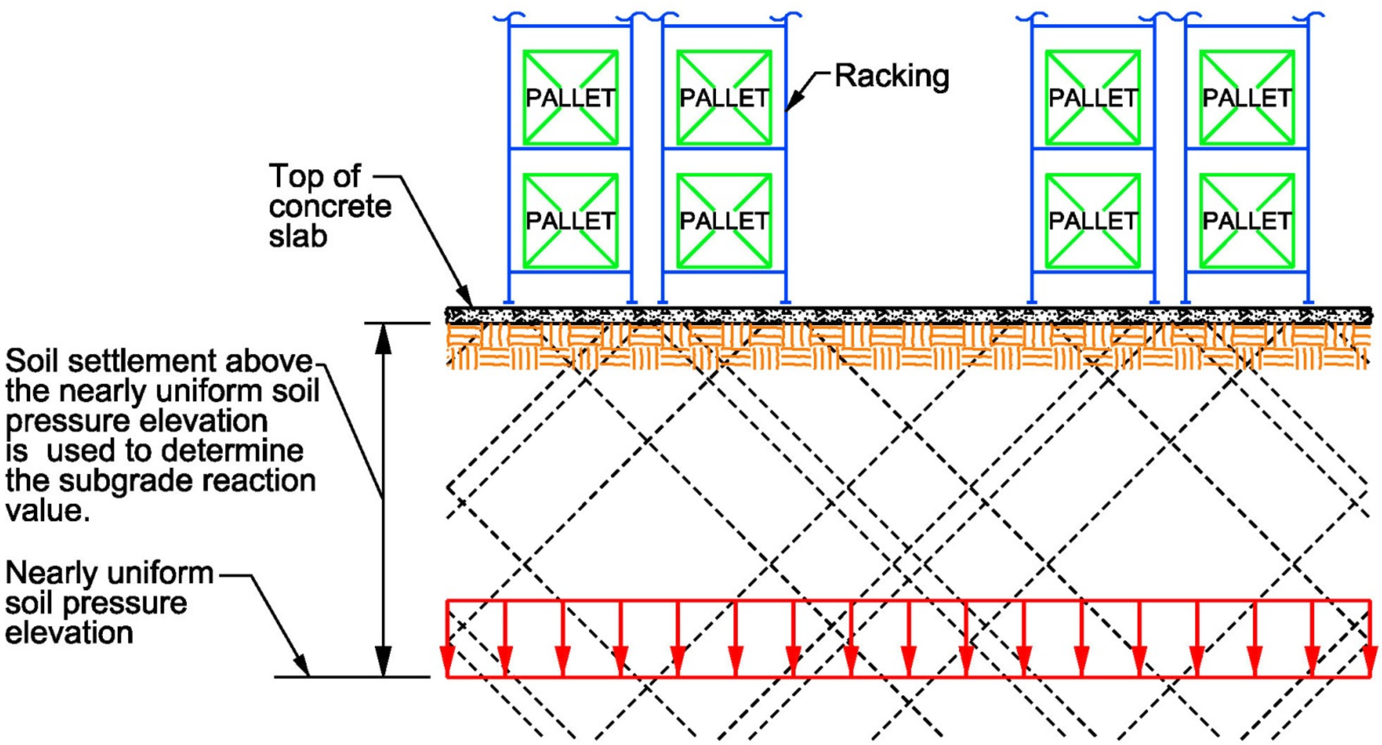


Fig. 3 Soil settlements used for evaluating subgrade reaction for wide area long-term loads

For soil settlements occurring below the nearly uniform pressure line, the entire slab moves down causing little influence on the differential deflections of the slab, and therefore the slab moments are not significantly affected. Some papers and text books have provided linear relationships of subgrade reaction with the bearing pressure used for foundation design, but these relationships would not be entirely correct because these relationships include the soil settlements below the nearly uniform pressure line. Additionally, using a thin layer of granular aggregate or cement treated base material to improve or increase the subgrade reaction for wheel loadings will only have a minimal effect for wide area sustained loadings because the deeper layers of soil are affected more.

To provide reasonable approximations of subgrade reactions for wide area sustained loadings, we have used two computer models as shown in Fig. 5. Fortunately, the slab stress analysis is not sensitive to the value of the subgrade reaction (large changes to the subgrade reaction only causes small changes to the slab's stresses); therefore, a reasonably approximate subgrade reaction is all that is needed for slab designs. The first computer model uses solid finite elements, which by the nature of the analysis will only produce stresses in the slab for settlements above the nearly uniform pressure elevation. The second computer model will use an equivalent subgrade reaction that will produce the same maximum stresses in the slab. By equating these two computer models, the equivalent subgrade reaction will have the reduction due to the wide area sustained loading.

The following is an example to illustrate our approach for determining the subgrade reaction for wide area sustained loadings. Fig. 4 shows the typical rack configuration of 4 ft. by 8 ft. with a 1 ft. flue space and an 8 ft. wide aisle used for our analysis. Aisle spacing can vary depending on the type of storage racking systems used. A storage racking system using a conventional lift truck commonly has aisle widths of 10 to 11 ft. A storage racking system that uses a narrow aisle racking system typically has aisle widths of 5 to 6 ft. As discussed above, only a reasonably approximate subgrade reaction is needed for slab design; therefore, we used an 8 ft. aisle width to approximate the two different storage racking system aisle widths.

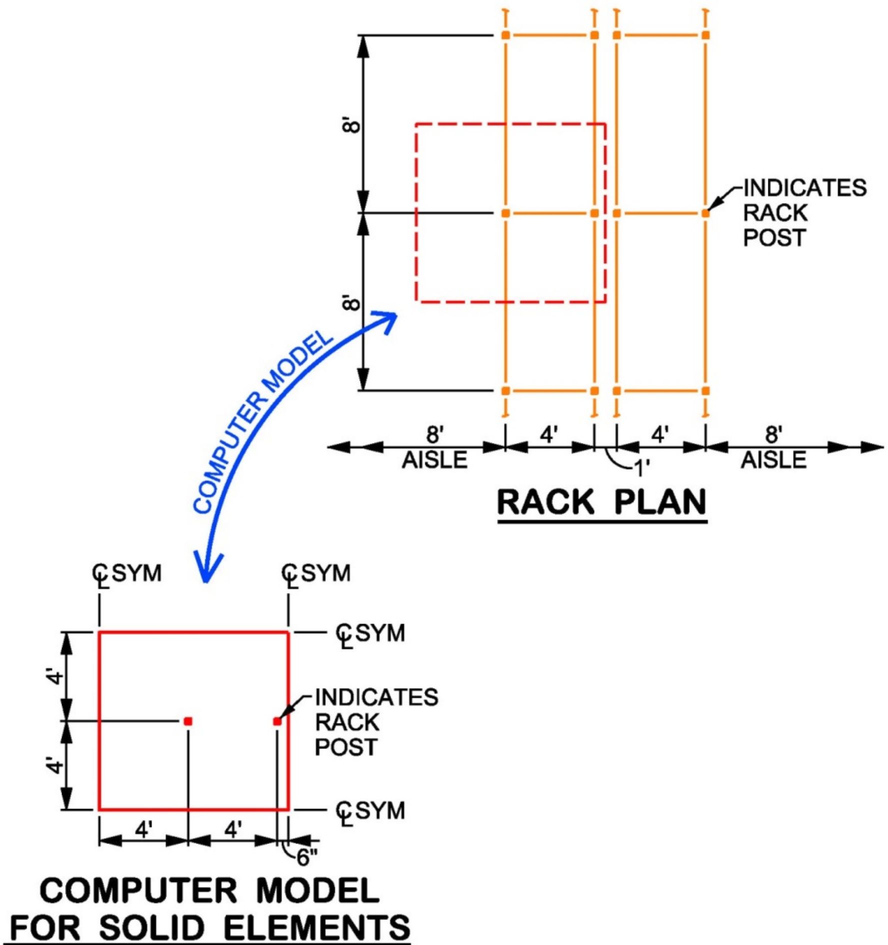


Fig. 4 Racking used for the computer model and symmetry boundaries

Fig. 5 shows the two computer models side-by-side. We used a 6" thick concrete slab supporting a rack post load of 7,000 pounds per post, which is a common long-term sustained loading for a distribution warehouse facility. Table 1 shows the soil's elastic moduli E_S and Poisson's ratios μ used for the different soil types. These soil values have a wide variation and are a composite from Ref.'s 5, 6 and 17. Fortunately, the way the analysis is performed by equating the two different models, only reasonable approximations of these soil values are needed.

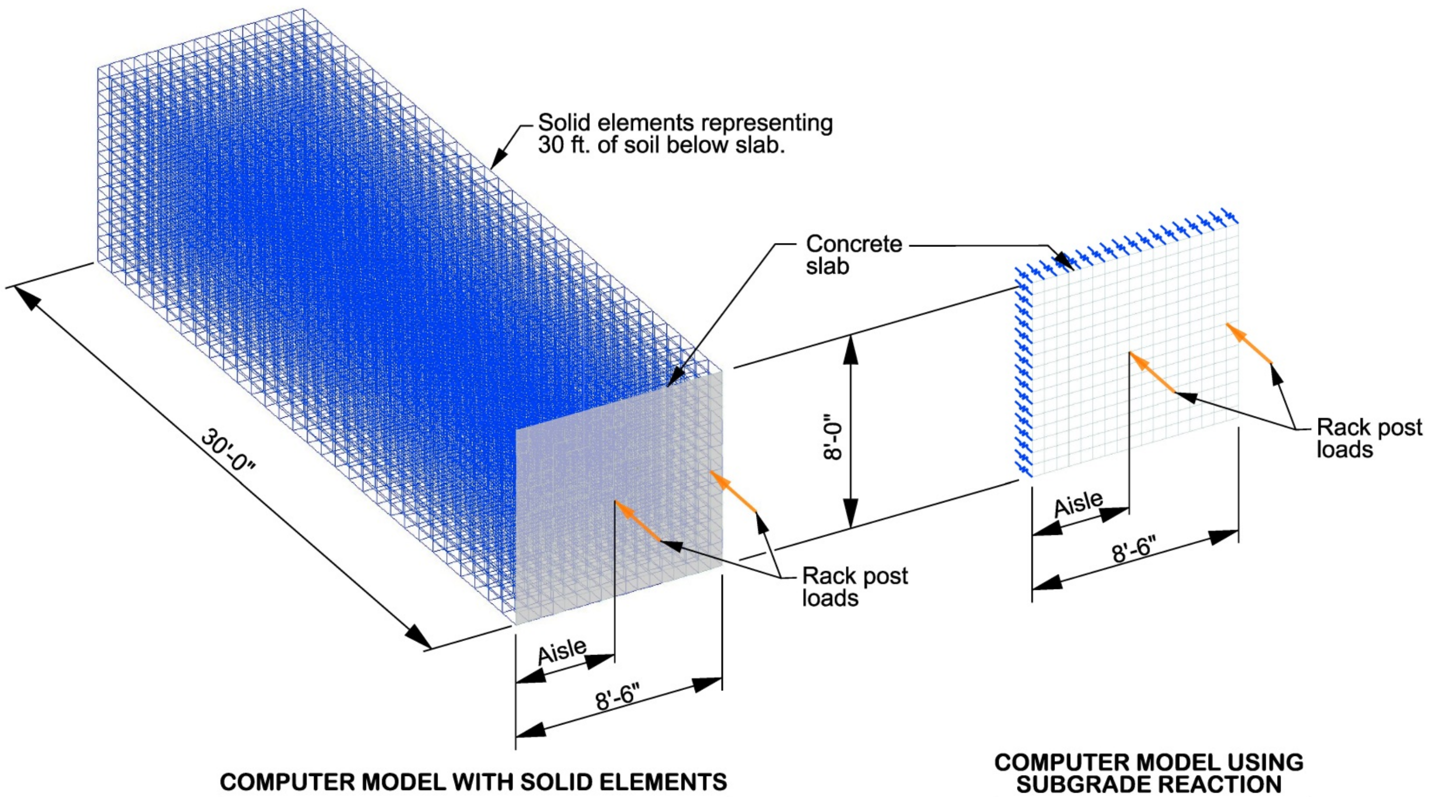
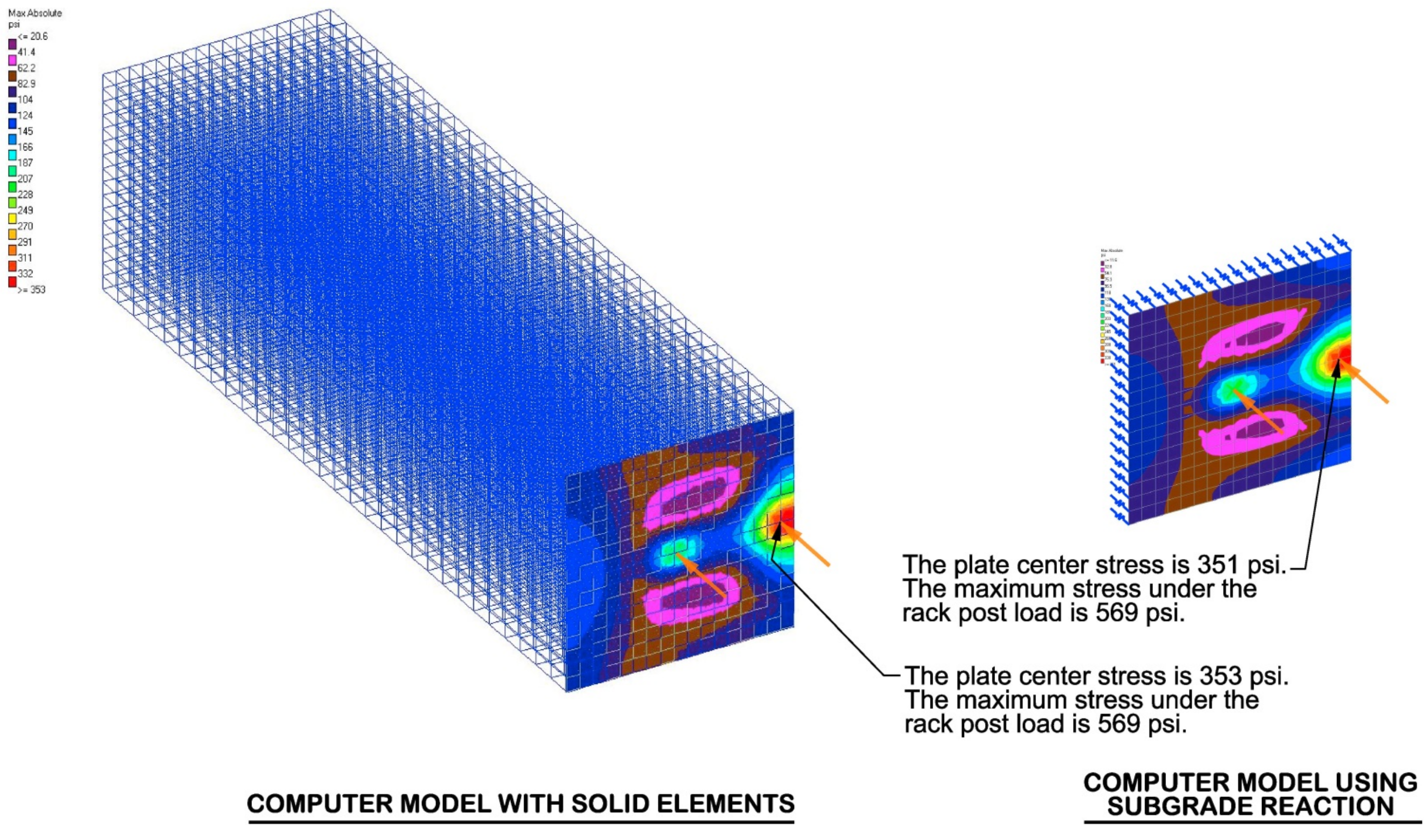


Fig. 5 Side-by-side computer models

For our example, we used a medium stiff clay with the soil modulus and Poisson's ratio values shown in Table 1. To approximate the long-term soil consolidation for the clay soils, we used ½ of the soil modulus. For our example, that would provide a value of 2,400 psi (½ of 4,800 psi). Using ½ of the soil modulus would approximately estimate the soil consolidation settlement to be nearly equal to the initial elastic settlement. For sandy soils, typically there are only initial elastic settlements and little long-term consolidation settlements. Therefore, long-term soil consolidation settlements were not considered for the sandy soils (the full elastic soil modulus was used in the analysis for sandy soils). These soil assumptions should be reviewed by the project geotechnical engineer and modified if needed for the site specific soil conditions.

By running several computer iterations, the equivalent subgrade reaction that produces the same maximum slab stress as the solid element model can be determined. For our example, the equivalent modulus of subgrade reaction was determined to be 80 pci. The 80 pci would have both the reduction of wide area loading and the reduction for long-term soil consolidation. The 80 psi would be 35% (80 pci/230 pci X 100%) of the plate subgrade reaction value. This reduction is in the range of other published reductions (Ref. 9, P. 13). The slab stresses are shown in Fig. 6.



Note that the center plate stress values and pattern are close to being the same. However, because the springs in the subgrade reaction model are uncoupled the stress pattern and center plate stresses will not be exactly the same as the computer model with solid elements.

Fig. 6 Side-by-side computer models showing the slab stresses

As discussed above, the nearly uniform soil pressure location for this model is approximately 14 ft. below the slab as shown in Fig. 7. In other words, the settlements in the 14 ft. of soil below the slab are the only ones that significantly affect the slab's stresses.

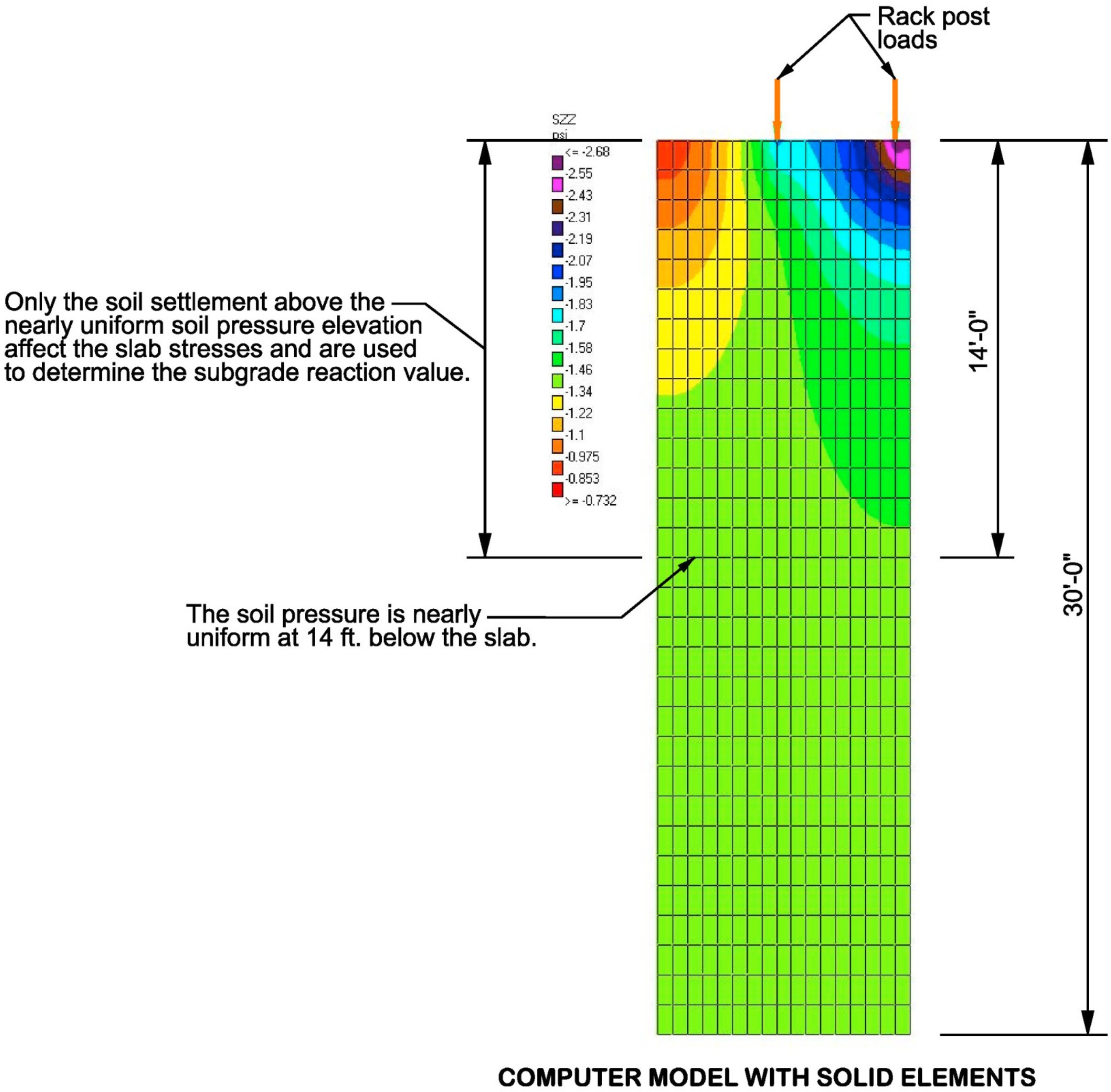


Fig. 7 Computer model showing soil stresses

Fig. 8 shows the computer models side-by-side slab deflections. The computer model using the solid finite elements will provide the total settlements for the loading, which has a maximum deflection of 0.152 in. The computer model using the subgrade reaction only provides the differential deflection from the slab's top surface and has a maximum deflection of 0.033 in. Notice that the differential deflection of 0.031 in. and 0.024 in are nearly the same for both models, which correspondingly provides the same slab maximum stresses for both computer models. The differential deflections will not be exactly the same because the springs in the subgrade reaction model are uncoupled.

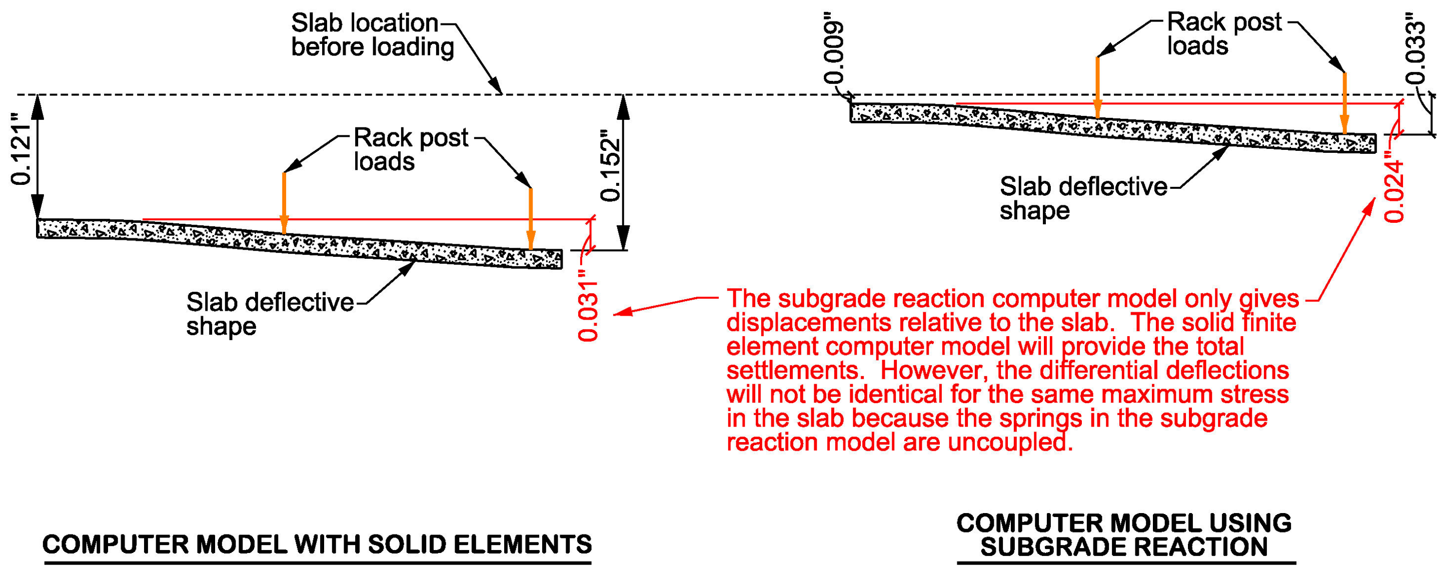


Fig. 8 Side-by-side computer models showing the slab's deflections

To determine the plate subgrade reaction K_p as shown in Fig. 1 above, the equation (Ref. 15, P. 408) for a concentrated load applied to a circular plate on a semi-infinite solid can be used.

$$\Delta = \frac{P(1-\mu^2)}{2RE_s} \quad EQ.1$$

Where:

P = Load applied to the plate
Δ = Deflection of the circular plate
R = Radius of the plate
E_s = Soil's elastic modulus
μ = Poisson's ratio

Substituting the following two equations EQ. 2 and EQ. 3 into EQ. 1:

$$K_p = \frac{\sigma}{\Delta} \quad EQ.2$$

$$\sigma = \frac{P}{\pi R^2} \quad EQ.3$$

Where:

K_p = Plate subgrade reaction
σ = Average soil pressure below the plate
Δ = Deflection of the circular plate

And writing the equation in terms of K_p provides:

$$K_p = \frac{2\,E_s}{\pi R(1-\mu^2)} \quad EQ.4$$

It can be clearly seen from EQ. 4 that the subgrade reaction is not a defined value but depends on both the soil type and area loaded. This equation was used to determine plate subgrade modulus values in Table 1.

These values are shown for information only and should not be used for slab designs with these types of soils. The K_{ws} values should be used for these types of soils.

Table 1 – Subgrade Reaction Design Values

Soil Type	E_s (psi)	μ	K_p (PCI)	K_w (PCI)	Ratio K_w/K_p	K_{ws} (PCI	Ratio K_{ws}/K_p
Clay Soft	2,000	0.45	110	70	0.64	50	0.45
Medium	4,800	0.35	230	130	0.57	80	0.35
Hard	10,600	0.20	470	260	0.55	140	0.30
Sandy	15,600	0.25	710	390	0.55	200	0.28
Sand Silty	2,000	0.30	90	70	0.78	---	---
Loose	2,500	0.30	120	80	0.67	---	---
Dense	9,500	0.35	460	240	0.52	---	---

Plate subgrade reaction design values used for short-term loadings over small areas, such as vehicle wheel loads.

Subgrade reaction design values reduced for wide area storage loadings, such as uniform or rack loads. Typically, sandy soils have mostly initial elastic settlements and little long-term soil consolidation. Therefore, these values are not reduced for long-term soil consolidation.

Subgrade reaction design values reduced for wide area storage loadings, such as uniform or rack loads, and reduced due to long-term soil consolidation settlements typical of clay types of soils.

Table 1 - Notes:

1. K_p is the plate subgrade reaction value.
2. K_w is the subgrade reaction reduced for wide area loading.
3. K_{ws} is the subgrade reaction reduced for wide area loading and long-term soil settlement consolidation.

Conclusion

It is our hope that this paper will provide the much needed clarity regarding what value of subgrade reaction should be used for the different loading cases, how the value should be determined, and the joint responsibilities of the structural and geotechnical engineers for determining the proper subgrade value to be used.

As discussed above, the proper subgrade reaction is a function of several variables, and the values in Table 1 are only valid for the stated assumptions. These values may not be appropriate for mats, large/heavy equipment foundations, tank foundations, continuous beams, etc.

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